

# Notes: 7.1 Put on bright yellow paper!

## \*Reciprocal Identities:

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

→ *Leave a little space between each type of identity!!*

\*Quotient Identities:

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

→ *Leave a little space between each type of identity!!*

\*Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

Ⓞ  $(\sin\theta)^2 + (\cos\theta)^2 = 1$

$$\tan^2\theta + 1 = \sec^2\theta$$

→ *Leave a little space between each type of identity!!*

## Double Angle Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

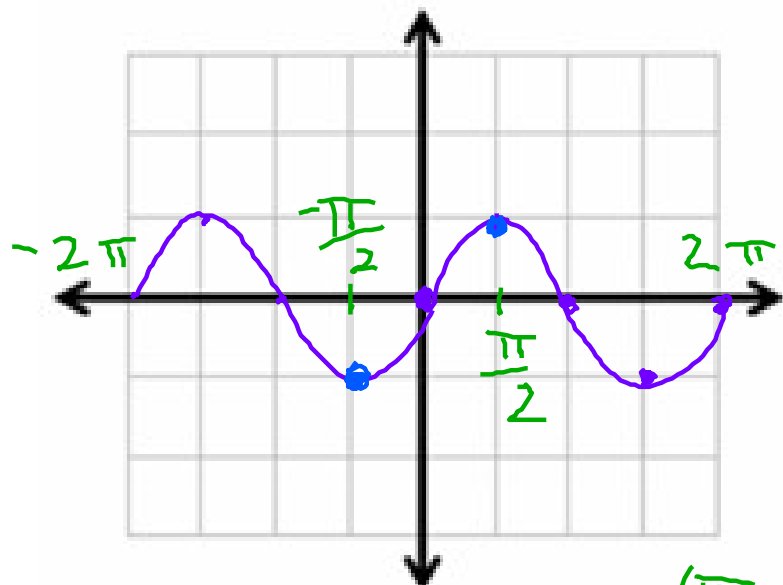
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\textcircled{\text{or}} = 1 - 2\sin^2\theta$$

$$\textcircled{\text{or}} = 2\cos^2\theta - 1$$

# \*Opposite Angle Identities:

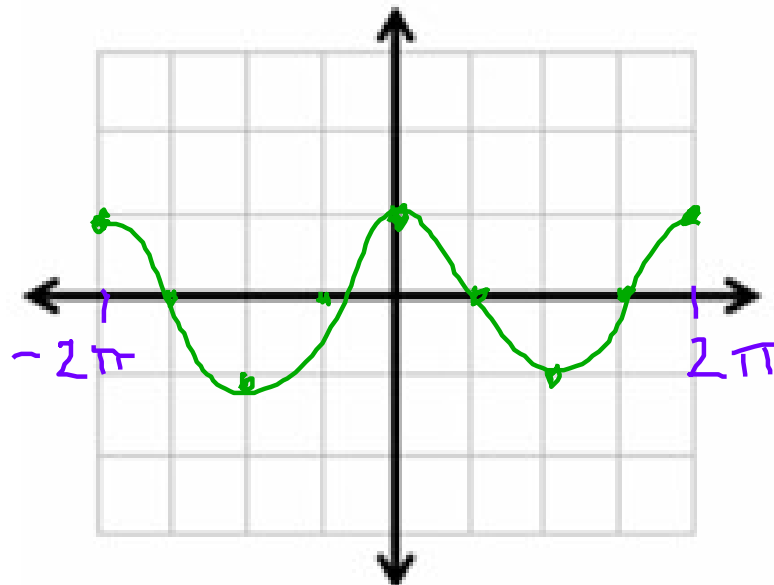
$$\sin(-\theta) = -\sin\theta$$



$$\sin\left(-\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right)$$

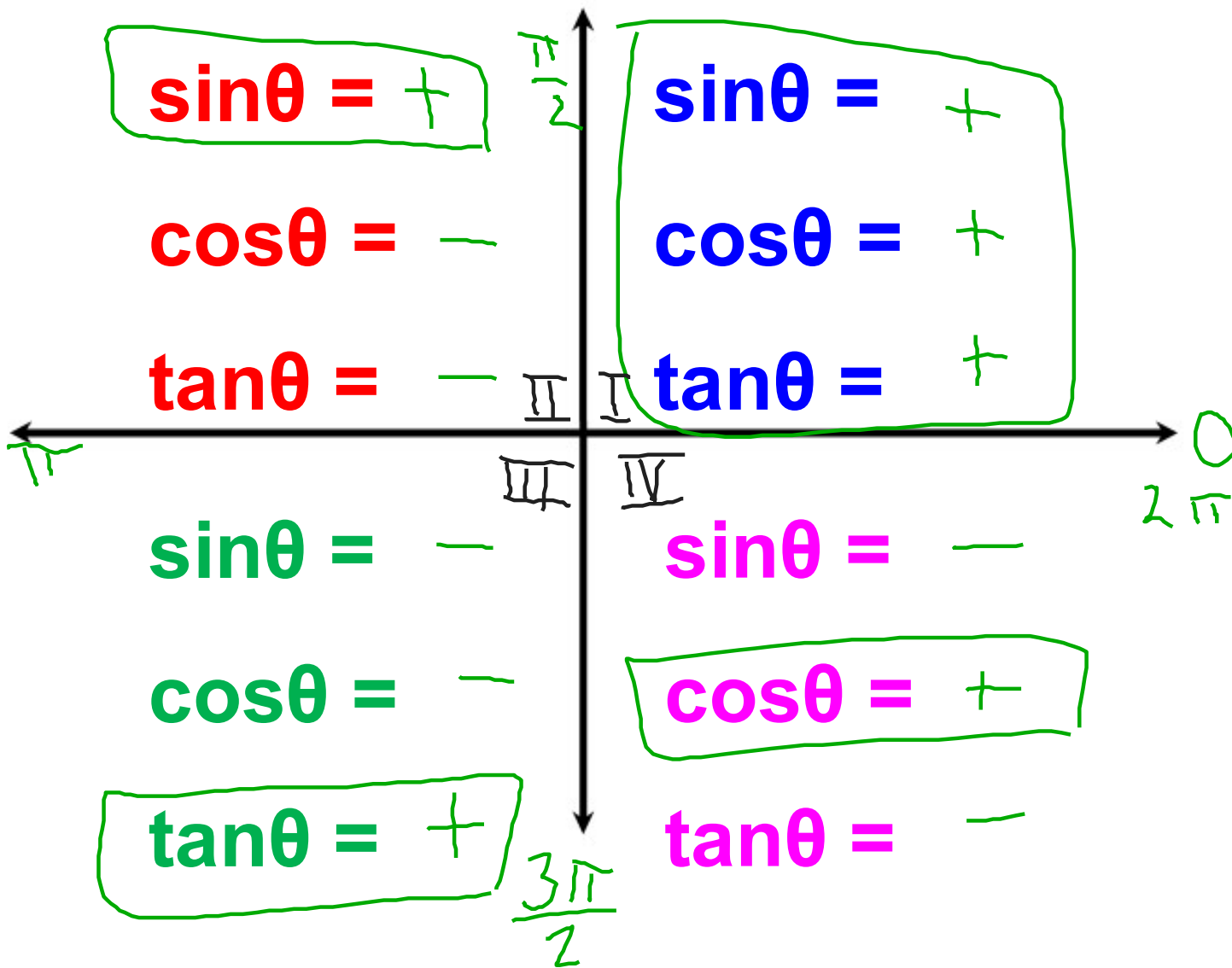
$$-1 = -1$$

$$\cos(-\theta) = \cos\theta$$



$$\cos(-2\pi) = \cos(2\pi)$$

$$1 = 1$$



From  
Ch. 5 & 6:

$\sin\theta = y$  or  $\frac{y}{r}$   
 $\cos\theta = x$  or  $\frac{x}{r}$   
 $\tan\theta = \frac{y}{x}$  or  $\frac{y}{x}$

unit circle  $\uparrow$   
 $r \neq 1$

# CH.7 INTRODUCTION TO IDENTITIES

Name:

Per:

#1-19: SHOW ALL WORK. YOU MUST USE IDENTITIES TO SOLVE EACH PROBLEM!

- a. Write identity
- b. Plug in values
- c. Show work and solve

Use the given information to determine the exact trigonometric value.

1.  $\cot \theta = -\frac{\sqrt{5}}{2}$ ,  $\frac{\pi}{2} < \theta < \pi$ ;  $\tan \theta$

2.  $\sin \theta = -\frac{1}{5}$ ,  $\pi < \theta < \frac{3\pi}{2}$ ;  $\cos \theta$

Simplify each expression.

3.  $\frac{\csc \theta}{\cot \theta}$

4.  $(\cos x)(\csc x)(\tan x)$

5.  $\frac{\sin 2\theta}{\cos \theta} \cdot \cot \theta$

## CHECK ANSWERS:

$$-\frac{3}{4} \quad -\frac{\sqrt{2}}{3} \quad -\frac{2\sqrt{5}}{5}$$

$$-\frac{2\sqrt{6}}{5} \quad -\frac{3\sqrt{13}}{13} \quad \frac{4\sqrt{3}}{3}$$

$$-\frac{2\sqrt{6}}{7} \quad \frac{\sqrt{2}}{4} \quad \frac{\sqrt{5}}{3} \quad \frac{\sqrt{15}}{4}$$

$$\sin x + \cos x \quad 1 \quad 2$$

$$\cos x \quad \csc x$$

$$\csc \theta \quad \sec \theta$$

$$2\cot \theta \quad 2\cos \theta$$

# Determine an exact value using identities:

1.  $\cot \theta = -\frac{\sqrt{5}}{2}$   
given

$$\frac{\pi}{2} < \theta < \pi$$

II

find  $\tan \theta = ?$

$\tan \theta = -$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$= \frac{1}{-\frac{\sqrt{5}}{2}} = -\frac{2}{\sqrt{5}}$$

$$= \boxed{-\frac{2\sqrt{5}}{5}}$$

## CHECK ANSWERS:

$$-\frac{3}{4} \quad -\frac{\sqrt{2}}{3} \quad -\frac{2\sqrt{5}}{5}$$

$$-\frac{2\sqrt{6}}{5} \quad -\frac{3\sqrt{13}}{13} \quad \frac{4\sqrt{3}}{3}$$

$$-\frac{2\sqrt{6}}{7} \quad \frac{\sqrt{2}}{4} \quad \frac{\sqrt{5}}{3} \quad \frac{\sqrt{15}}{4}$$

$$\sin x + \cos x \quad 1 \quad 2$$

$$\cos x \quad \csc x$$

$$\csc \theta \quad \sec \theta$$

$$2\cot \theta \quad 2\cos \theta$$



### \*Reciprocal Identities:

$$\sin\theta = \frac{1}{\csc\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

### Double Angle Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\textcircled{\text{or}} = 1 - 2\sin^2\theta$$

$$\textcircled{\text{or}} = 2\cos^2\theta - 1$$

### \*Quotient Identities:

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \quad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

### \*Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \quad 1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

### \*Opposite Angle Identities:

$$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta$$

# Determine an exact value using identities:

2.  $\sin \theta = -\frac{1}{5}$        $\pi < \theta < \frac{3\pi}{2}$       find  $\cos \theta$

III       $\cos = -$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(-\frac{1}{5}\right)^2 + \cos^2 \theta = 1$$

$$\frac{1}{25} + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \frac{1}{25}$$

$$\sqrt{\cos^2 \theta} = \sqrt{\frac{24}{25}}$$

$$\cos \theta = -\frac{\sqrt{24}}{5}$$

## CHECK ANSWERS:

$$-\frac{3}{4} \quad -\frac{\sqrt{2}}{3} \quad -\frac{2\sqrt{5}}{5}$$

$$-\frac{2\sqrt{6}}{5} \quad -\frac{3\sqrt{13}}{13} \quad \frac{4\sqrt{3}}{3}$$

$$-\frac{2\sqrt{6}}{7} \quad \frac{\sqrt{2}}{4} \quad \frac{\sqrt{5}}{3} \quad \frac{\sqrt{15}}{4}$$

$$\sin x + \cos x \quad 1 \quad 2$$

$$\cos x \quad \csc x$$

$$\csc \theta \quad \sec \theta$$

$$2\cot \theta \quad 2\cos \theta$$

Simplify each expression:

$$\begin{aligned} 3. \quad \frac{\csc \theta}{\cot \theta} &= \frac{\frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}} = \frac{1}{\cancel{\sin \theta}} \cdot \frac{\cancel{\sin \theta}}{\cos \theta} \\ &= \frac{1}{\cos \theta} \\ &= \boxed{\sec \theta} \end{aligned}$$

4.  $(\cos x)(\csc x)(\tan x)$

CHECK ANSWERS:

$$-\frac{3}{4} \quad -\frac{\sqrt{2}}{3} \quad -\frac{2\sqrt{5}}{5}$$

$$-\frac{2\sqrt{6}}{5} \quad -\frac{3\sqrt{13}}{13} \quad \frac{4\sqrt{3}}{3}$$

$$-\frac{2\sqrt{6}}{7} \quad \frac{\sqrt{2}}{4} \quad \frac{\sqrt{5}}{3} \quad \frac{\sqrt{15}}{4}$$

$$\sin x + \cos x \quad 1 \quad 2$$

$$\cos x \quad \csc x$$

$$\csc \theta \quad \cancel{\sec \theta}$$

$$2\cot \theta \quad 2\cos \theta$$

Simplify each expression:

$$5. \frac{\sin 2\theta}{\cos \theta} \cdot \cot \theta$$

$$\frac{2\cancel{\sin \theta} \cancel{\cos \theta}}{\cancel{\cos \theta}} \cdot \frac{\cancel{\cos \theta}}{\cancel{\sin \theta}} = 2\cos \theta$$

CHECK ANSWERS:

$$-\frac{3}{4} \quad -\frac{\sqrt{2}}{3} \quad -\frac{2\sqrt{5}}{5}$$

$$-\frac{2\sqrt{6}}{5} \quad -\frac{3\sqrt{13}}{13} \quad \frac{4\sqrt{3}}{3}$$

$$-\frac{2\sqrt{6}}{7} \quad \frac{\sqrt{2}}{4} \quad \frac{\sqrt{5}}{3} \quad \frac{\sqrt{15}}{4}$$

$$\sin x + \cos x \quad 1 \quad 2$$

$$\cos x \quad \csc x$$

$$\csc \theta \quad \sec \theta$$

$$2\cot \theta \quad \cancel{2\cos \theta}$$

**CLEARLY SHOW ALL WORK ON A SEPARATE SHEET OF PAPER OR ON THE BACK OF THIS PAPER.**

Use the given information to determine the exact trigonometric value.

6.  $\sin \theta = \frac{\sqrt{3}}{4}$ ,  $0 < \theta < \frac{\pi}{2}$ ;  $\csc \theta$

7.  $\sin \theta = \frac{1}{4}$ ,  $0 < \theta < \frac{\pi}{2}$ ;  $\cos \theta$

8.  $\cos \theta = -\frac{2}{3}$ ,  $\frac{\pi}{2} < \theta < \pi$ ;  $\sin \theta$

9.  $\csc \theta = \frac{\sqrt{11}}{3}$ ,  $\frac{\pi}{2} < \theta < \pi$ ;  $\cot \theta$

10.  $\sec \theta = -\frac{5}{4}$ ,  $\frac{\pi}{2} < \theta < \pi$ ;  $\tan \theta$

11.  $\sin \theta = -\frac{1}{3}$ ,  $\pi < \theta < \frac{3\pi}{2}$ ;  $\tan \theta$

12.  $\tan \theta = \frac{2}{3}$ ,  $\pi < \theta < \frac{3\pi}{2}$ ;  $\cos \theta$

13.  $\sec \theta = -\frac{7}{5}$ ,  $\pi < \theta < \frac{3\pi}{2}$ ;  $\sin \theta$

Simplify each expression.

14.  $\frac{\sec x}{\tan x}$

15.  $\frac{\cot \theta}{\cos \theta}$

16.  $(\csc^2 \theta)(\sin 2\theta)$

17.  $\cos x \cdot \tan x + \sin x \cdot \cot x$

18.  $\sin x \cdot \cos x \cdot \sec x \cdot \cot x$

19.  $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

*hint: square each part, then combine like terms*  
*reminder:  $(x + y)^2 = x^2 + 2xy + y^2$*